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SOLUTION OF LIMITING PROBLEMS OF EXPANDING FLOWS OF GASEOUS
SUSPENSIONS

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UDC 532.529.5

The equations of equilibrium expanding flows of a gaseous suspension with an arbitrary volume concentration of particles are transformed into the equations of an expanding ideal gas and, in particular, in the two-dimensional stationary case, into the linear Chaplygin equations.

Studies of limiting motions, for example motions with equal velocities of the soil and of the gas expanding into its pores, or equilibrium flows, when not only the velocities but also the temperatures are equal, help to elucidate the important qualitative features of the flows of dispersed media and sometimes permit obtaining results with satisfactory accuracy.

The motion of an equilibrium mixture is described, generally speaking, by the system of equations for a single-phase, continuous, imperfect-gas medium (see, for example, [1]). The perfect-gas equations are obtained from this system only in the case of a low volume concentration of particles and in the absence of phase transitions, which permits applying the analytical apparatus of classical gas dynamics [1, 2].

The results of this work follow from the representations of the mechanism of the phenomena given by S. A. Khristianovich in his research on the properties of dispersed flows for the example of nonstationary one-dimensional flows of soil and gas contained in its pores [3, 4].

We study below motion in a space with interphase heat exchange. For large volume concentrations of particles, we restrict ourselves to expansion flows $d\rho/dt$.

In gas dynamics, an example of an expanding flow is the motion of a gaseous suspension in a nozzle.

The equations of an ideal perfect pseudogas with an arbitrary volume concentration of particles, in particular, in the two-dimensional stationary case — the linear Chaplygin equations, are obtained for describing the limiting states of expanding flows of gaseous suspensions.

Institute of Problems in Mechanics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 1, pp. 29-35, January, 1985. Original article submitted September 12, 1983.

The distribution of particles is assumed to be uniform, and the motion of the gaseous suspension is assumed to be free, i.e., without the transmission of forces via contacts between particles.

The system of equations of motion of gaseous suspensions for an arbitrary volume concentration of particles has the following form:

$$\frac{D}{Dt} \ln(1-m)\rho_s + \operatorname{div} \mathbf{w} = 0, \quad (1)$$

$$\frac{d}{dt} \ln \rho m + \operatorname{div} \mathbf{v} = 0, \quad (2)$$

$$(1-m)\rho_s \frac{D\mathbf{w}}{Dt} = \mathbf{F}, \quad (3)$$

$$\rho m \frac{d\mathbf{v}}{dt} = -\operatorname{grad} pm - \mathbf{F}, \quad (4)$$

$$p = R\rho T, \quad (5)$$

$$\frac{d}{dt} \ln(\rho \rho^{-\alpha} m^{1-\alpha}) = \frac{\alpha(\alpha-1)}{a^2} A(\mathbf{v}-\mathbf{w}) \frac{D\mathbf{w}}{Dt} - \frac{A}{T} \frac{c_s}{c_v} \frac{DT_s}{Dt}, \quad (6)$$

$$\frac{c_s \rho_s d}{6} \frac{DT_s}{Dt} = -\alpha(T_s - T), \quad (7)$$

$$A = \frac{(1-m)\rho_s}{\rho m}; \quad \mathbf{F} = -p \operatorname{grad} m + \mathbf{f}. \quad (8)$$

The first term on the right side of the second formula of (8) appears due to the change in the porosity, while the second term \mathbf{f} is expressed by different formulas depending on the conditions of the problem, for example,

$$\mathbf{f} = \frac{\mu}{k} (\mathbf{v}-\mathbf{w}) \quad (m \ll 1); \quad \mathbf{f} = 18 \frac{\mu}{d^2} (1-m)(\mathbf{v}-\mathbf{w}) \quad (1-m \ll 1), \quad (9)$$

where the first formula is the analog of Darcy's law and the second follows from Stokes' law [3, 4]. The coefficient k has the dimensions of permeability and is on the order of $O(d^2)$.

Equations (3) and (6) are valid at the interior points of the flow region, since at the boundary the action of the wall must also be taken into account. Equation (6), proposed by S. A. Khristianovich in [3, 4] for the case of a one-dimensional motion with $\alpha = 0$, is the transformed equation of balance of the energy of the gaseous phase:

$$\rho m \left(\mathbf{v} \frac{d\mathbf{v}}{dt} + c_v \frac{dT}{dt} \right) = -\mathbf{F}\mathbf{w} - \operatorname{div} pm\mathbf{v} - (1-m)\rho_s c_s \frac{DT_s}{Dt} \quad (10)$$

and is obtained by subtracting from (10) the scalar product of Eq. (4) with \mathbf{v} and using the formula $pm \operatorname{div} \mathbf{v} = \operatorname{div} pm\mathbf{v} - \mathbf{v} \operatorname{grad} pm$ and Eqs. (3), (4), and (5). The classical formula $p/\rho^\alpha = \text{const}$ follows from Eq. (6) in the limits $m \rightarrow 1$, $A \rightarrow 0$. If, on the other hand, for $1-m \ll 1$ the quantity A is not small because $\rho \ll \rho_s$, then in the limit, setting $m = 1$ on the left side of Eq. (6), we obtain an equation for a gaseous suspension with negligibly small volume concentration, but significant mass concentration of particles.

The motion of the medium will be close to equilibrium for sufficiently small particles, since according to (7) and (9) $\mathbf{v} = \mathbf{w}$, $T_s = T$ when $d = 0$. Then, in the limit it follows from Eqs. (1), (8), and (6) that

$$A = \frac{(1-m)\rho_s}{\rho m} = \text{const}; \quad \frac{d}{dt} \ln(\rho \rho^{-\alpha} m^{1-\alpha}) = -\varepsilon \frac{d}{dt} \ln T, \quad (11)$$

where $\varepsilon = Ac_s/c_v$ with $T_s = T$ and $\varepsilon \equiv 0$ with $\alpha = 0$, $A \neq 0$.

The case $\varepsilon \equiv 0$ corresponds to motion which is at equilibrium with respect to velocity, but with a "frozen" particle temperature $T_s = \text{const}$, as, for example, the motion of grains of soil, where heat exchange with the gas expanding in its pores is negligibly small.

It follows from Eqs. (4), (5), and (6) that the specific entropy of the gaseous phase $s = c_v \ln(p/\rho^\alpha)$ changes according to the equation

$$\frac{ds}{dt} = R \frac{d}{dt} \ln m + \frac{v-w}{\rho m} \frac{F}{T} - \frac{Ac_s}{T} \frac{DT_s}{Dt} \quad (12)$$

and for this reason the change in the specific entropy of the equilibrium mixture s_1 is equal to

$$\frac{ds_1}{dt} = \frac{1}{1+A} \left(\frac{ds}{dt} + A \frac{c_s}{T} \frac{dT}{dt} \right) = \frac{R}{1+A} \frac{d}{dt} \ln m. \quad (13)$$

The final result for ds_1/dt is also valid with $\epsilon \equiv 0$, $T_s = \text{const.}$ Based on (11), $dm = -Am^2 D(\rho/\rho_s)$.

Therefore, for a mixture described by the given system of equations, the entropy of equilibrium and close to equilibrium flows increases only with $d\rho/dt < 0$.

As S. A. Khristianovich showed, for compression flows $d\rho/dt > 0$ the equation of energy of the gaseous phase differs fundamentally from Eqs. (10) or (6) [4]. This difference is, in particular, associated with the necessity of taking into account the additional work performed by the external forces $-pDm/Dt$ as a result of the change in m accompanying compression.

We shall restrict ourselves to the system of equations (1)-(8), i.e., to flows for which $d\rho/dt < 0$. We note that for $1-m \ll 1$, $\rho \ll \rho_s$ and any A , from system (1)-(8) we can also obtain approximate solutions for flows with $d\rho/dt > 0$, because the motion of such a mixture will be mathematically close to the limit $A \neq 0$ with $m \rightarrow 1$, $\rho/\rho_s \rightarrow 0$, for which the equations of the energy of the gaseous phases with $d\rho/dt < 0$ and $d\rho/dt > 0$ coincide [4].

After adding Eqs. (3) and (4) with $v=w$ and from (11) we obtain

$$-\frac{\text{grad } pm}{(1+A)\rho m} = \frac{\partial v}{\partial t} + \frac{1}{2} \text{grad } v^2 - v \times \text{rot } v, \quad (14)$$

$$pm/(\rho m)^{\frac{\kappa+\epsilon}{1+\epsilon}} = \text{const.}$$

In the variables $p_1 = pm$, $\rho_1 = (1+A)\rho m$ we arrive at the equations of an ideal adiabatic flow of gas with the adiabatic index κ_1 and the gas constant R_1 :

$$\kappa_1 = c_{p_1}/c_{v_1} = \frac{\kappa + \epsilon}{1 + \epsilon}, \quad R_1 = \frac{R}{1+A}, \quad c_{v_1} = \frac{c_v(1+\epsilon)}{1+A}. \quad (15)$$

The parameters introduced are physically well founded. A unit of mass of the mixture consists of a fraction $1/(1+A)$ of gas and a fraction $A/(1+A)$ of particles. The heat capacity of the mixture c_{v_1} must be equal to the sum of $c_v/(1+A)$ for the gas and $Ac_s/(1+A)$ for the particles, which also follows because of $\epsilon = Ac_s/c_v$ from Eq. (15). The fact that c_{p_1} differs from c_{v_1} is attributable to the presence of the gas phase. In the limit $A \rightarrow \infty$, i.e., as the gas vanishes ($m \rightarrow 0$), $c_{p_1} \rightarrow c_{v_1} \rightarrow c_s$. In the case of particles with a "frozen" temperature ($\epsilon \equiv 0$), the values of c_{v_1} and c_{p_1} are determined only by the $1/(1+A)$ part due to the gas phase.

We shall follow based on the formulas and equations of gas dynamics the relationship between the different parameters of the flows of pseudogas, i.e., an equilibrium mixture and its gas phase. Based on (14) with $\text{rot } v = 0$ we obtain the Cauchy integral

$$\int \frac{dp_1}{\rho_1} + \frac{1}{2} v^2 + \frac{\partial \varphi}{\partial t} = \chi(t); \quad v = \text{grad } \varphi. \quad (16)$$

For stationary flows $\partial \varphi / \partial t \equiv 0$, $\chi(t) = \text{const.}$ Then from (14) and (16), keeping in mind (in view of $d\rho/dt < 0$) the formality of the retardation parameters, we obtain

$$p_1/p_{10} = (\rho_1/\rho_{10})^{\kappa_1}, \quad (17)$$

$$\frac{\rho_1}{\rho_{10}} = \left(1 - \frac{\kappa(\kappa-1)(1+A)}{2(\kappa+\epsilon)} \frac{v^2}{a_0^2} \right)^{\frac{1+\epsilon}{\kappa-1}} = \left(1 - \frac{\kappa_1-1}{2} \frac{v^2}{a_{10}^2} \right)^{\frac{1}{\kappa_1-1}}$$

or in a different form

$$\frac{\rho_1}{\rho_{1\infty}} = \left(1 - \frac{\kappa(\kappa-1)(1+A)}{2(\kappa+\varepsilon)} \frac{v^2 - v_\infty^2}{a_\infty^2} \right)^{\frac{1+\varepsilon}{\kappa-1}} = \left(1 - \frac{\kappa_1-1}{2} \frac{v^2 - v_\infty^2}{a_{1\infty}^2} \right)^{\frac{1}{\kappa_1-1}}. \quad (18)$$

Using $a^2 = \kappa p/\rho$, $a_1^2 = \kappa_1 p_1/\rho_1$ and Eq. (17), we find

$$a^2 = \beta a_1^2; \quad a^2 = a_0^2 - \frac{\kappa(\kappa-1)(1+A)}{2(\kappa+\varepsilon)} v^2; \quad a_1^2 = a_{10}^2 - \frac{\kappa_1-1}{2} v^2; \quad (19)$$

$$a_*^2 = \left(1 + \frac{2\varepsilon}{\kappa+1} \right) \gamma a_{1*}^2;$$

$$\beta = \frac{\kappa(1+\varepsilon)(1+A)}{\kappa+\varepsilon} \geq 1; \quad \gamma = \frac{1+A}{1+A/h^2 + 2\varepsilon/\kappa(\kappa+1)}.$$

Instead of v_∞ , a_∞ , $a_{1\infty}$ we can take the values of these quantities at any other point in the flow region. If $1-m \ll 1$, then $p_1 \approx p$, $\rho_1 \approx (1+A)\rho$. Equations (17)-(19) then coincide with those obtained previously in the study of motions in application to nozzles (see, for example, [5]).

The magnitude of the volume concentration of particles $1-m$, based on (11), (18), and (19), can be determined from the formula

$$\frac{1-m}{1-m_\infty} = \left(\frac{1-\gamma\lambda^2/h^2}{1-\gamma\lambda_\infty^2/h^2} \right)^{\frac{1+\varepsilon}{\kappa-1}} = \left(\frac{1-\lambda_1^2/h_1^2}{1-\lambda_{1\infty}^2/h_1^2} \right)^{\frac{1}{\kappa_1-1}}, \quad (20)$$

where

$$\lambda = \frac{v}{a_*}; \quad \lambda_1 = \frac{v}{a_{1*}} = \sqrt{\left(1 + \frac{2\varepsilon}{\kappa+1} \right) \gamma \lambda}; \quad (21)$$

$$h_1^2 = \frac{\kappa_1+1}{\kappa_1-1}; \quad \frac{\lambda_1^2}{h_1^2} = \frac{\kappa_1-1}{2} \frac{v^2}{a_{10}^2}.$$

Mach's number of the gas phase $M = v/a$ and of the pseudogas $M_1 = v/a_1$ are related to λ and λ_1 by the formulas

$$M^2 = \frac{2(\kappa+\varepsilon)\lambda^2}{2(\kappa+\varepsilon) + \kappa(\kappa-1)(1+A)(1-\lambda^2)}; \quad (22)$$

$$M_1^2 = \frac{2}{\kappa_1+1} \frac{\lambda_1^2}{1-\lambda_1^2/h_1^2}; \quad M_1^2 = \beta M^2.$$

Because the equations governing the flows of the pseudogas and of the equilibrium mixture coincide, their characteristics and limiting lines will be the same.

According to (19), the velocity of sound a_1 in the pseudogas, i.e., the velocity of propagation of weak disturbances in the equilibrium mixture, is less than the velocity of sound in the gas phase. The higher the concentration of particles $1-m$, the longer will be the path of the weak disturbances propagating through the gas. The critical velocity of the pseudogas $v = a_{1*}$ is reached with a subsonic velocity of the gas $\lambda = \lambda_* = [(1+2\varepsilon/(\kappa+1))\gamma]^{-1/2}$, and therefore, $\lambda_1 > \lambda$.

Flows of high-concentration mixtures with $\varepsilon \neq 0$ and $\varepsilon = Ac_S/c_V$ are considerably different. Thus, for example, from the relations for $v_{\max}(a=0)$

$$v_{\max}^2 = v_\infty^2 + \frac{2(\kappa+\varepsilon)a_\infty^2}{\kappa(\kappa-1)(1+A)},$$

$$\lambda_{\max}^2 = \frac{h^2}{\gamma} \equiv 1 + \frac{2(\kappa+\varepsilon)}{\kappa(\kappa-1)(1+A)} \equiv \lambda_\infty^2 + \frac{2(\kappa+\varepsilon)[2(\kappa+\varepsilon) + \kappa(\kappa-1)(1+A)]}{\kappa(\kappa-1)(1+A)[2(\kappa+\varepsilon) + \kappa(\kappa-1)(1+A)M_\infty^2]}$$

we conclude that v_{\max} and α_* in the case $\varepsilon \equiv 0$ will be close to v_∞ at sufficiently large value of A. In other words, the smaller the mass of the gas compared to the mass of the particles, evidently, the less it can expand under the conditions $v=w$ and change the motion of the mixture.

The flow of a pseudogas with sufficiently large values of A will be hypersonic, i.e., $\lambda_{1\infty}$ is close to $\lambda_{1\max} = h$, and the velocity of sound $\alpha_1 \approx a_\infty/A^{1/2}$ is small. In the case $\varepsilon = Ac_S/c_V$, on the other hand, with A of the order of $O(10^2)$ and higher we already obtain values for v_{\max} close to the mathematical limit differing from v_∞ ($A \rightarrow \infty$):

$$v_{\max}^2 \rightarrow v_\infty^2 + \frac{2c_s a_\infty^2}{\kappa(\kappa-1)c_v}; \quad \lambda_{\max}^2 \rightarrow 1 + \frac{2c_s}{\kappa(\kappa-1)c_v};$$

$$\frac{\lambda_{1\max}^2}{\lambda_{1\infty}^2} = \frac{\lambda_{\max}^2}{\lambda_\infty^2} \rightarrow 1 + \frac{2c_s}{\kappa(\kappa-1)c_v M_\infty^2}.$$

Here an amount of gas which is small compared to the mass of the particles changes by a finite amount the velocity of the equilibrium mixture due to the transfer of thermal energy by the particles to the gas.

We note that actually the gas escapes from the pores in the soil and in order for the equilibrium model to correspond qualitatively to reality as the volume concentration of particles increases, the sizes of the particles must be substantially decreased.

To describe the motion of an equilibrium mixture, both the equation

$$\left(1 - \frac{\Phi_x^2}{a_1^2}\right) \Phi_{xx} + \left(1 - \frac{\Phi_y^2}{a_1^2}\right) \Phi_{yy} + \left(1 - \frac{\Phi_z^2}{a_1^2}\right) \Phi_{zz} - \frac{2\Phi_x\Phi_y}{a_1^2} \Phi_{xy} - \frac{2\Phi_x\Phi_z}{a_1^2} \Phi_{xz} - \frac{2\Phi_y\Phi_z}{a_1^2} \Phi_{yz} = 0,$$

which follows from the equation of continuity (2) using the formula $d \ln \rho_1 / dv^2 = \beta / 2a^2$ obtained from (17) and (19), and Chaplygin's system of equations- which is simply derived for two-dimensional motions from the conditions of total differentials

$$dx = \frac{\cos \theta}{v} d\varphi - \frac{\rho_{10}}{\rho_1 v} \sin \theta d\psi, \quad dy = \frac{\sin \theta}{v} d\varphi + \frac{\rho_{10}}{\rho_1 v} \cos \theta d\psi; \quad (23)$$

$$\frac{\partial \varphi}{\partial \theta} = \sqrt{K} \frac{\partial \psi}{\partial \sigma}, \quad \frac{\partial \varphi}{\partial \sigma} = -\sqrt{K} \frac{\partial \psi}{\partial \theta}, \quad (24)$$

where

$$\sqrt{K} = \sqrt{\frac{Q(\lambda)Q'(\lambda)}{P(\lambda)P'(\lambda)}}; \quad \sigma(\lambda) = \int_{\lambda_*}^{\lambda} \sqrt{\frac{P'(\lambda)Q'(\lambda)}{P(\lambda)Q(\lambda)}} d\lambda,$$

are valid. We have:

$$\sqrt{K} = \sqrt{\frac{1 - (1 + 2\varepsilon/(\kappa + 1))\gamma\lambda^2}{(1 - \gamma\lambda^2/h^2)^{\kappa^2 + 2\varepsilon/(\kappa-1)}}} = \sqrt{\frac{1 - \lambda_1^2}{(1 - \lambda_1^2/h_1^2)^{\kappa_1^2}}}, \quad (25)$$

$$\sigma = \int_{\lambda_*}^{\lambda} \sqrt{\frac{1 - (1 + 2\varepsilon/(\kappa + 1))\gamma\lambda^2}{1 - \gamma\lambda^2/h^2}} \frac{d\lambda}{\lambda} = \int_1^{\lambda_1} \sqrt{\frac{1 - \lambda_1^2}{1 - \lambda_1^2/h_1^2}} \frac{d\lambda_1}{\lambda_1}.$$

According to (25) and (23), the solution $\varepsilon \equiv 0$ of Eq. (24) with $\varphi(\sigma, \theta)$ given beforehand corresponds, because of $\kappa_1 = \kappa$, $h_1 = h$, in the x,y plane to the reduced A-invariant velocity field of the pseudogas $\lambda_1\theta$. For $\varepsilon = Ac_S/c_V$ Eqs. (24) depend on A via h_1^2 and to the solution $\varphi(\sigma, \theta)$ there corresponds in the x, y plane its own field $\lambda_1\theta$ for each A.

NOTATION

t, time; v and w, ρ and ρ_S , T and T_S , d/dt and D/Dt, respectively, velocity, density, temperature, and derivatives for the gas and the particles; p, pressure of the gas; R, gas constant; a , velocity of sound in the gas; α , coefficient of heat transfer from the particles to the gas; m and F, volume content of gas and the force exerted by the gas on the particles

per unit volume of the mixture; $i = F + \rho \text{grad} m$; d , particle diameter; c_v , c_p , and c_s , heat capacities of the gas and of the particles; μ , coefficient of viscosity of the gas, $\kappa = c_p/c_v$, $h^2 = (\kappa + 1)/(\kappa - 1)$; A , ratio of the reduced densities of the particles and of the gas; $\epsilon = Ac_s/c_p$ with $T_s = T$ and $\epsilon \equiv 0$ with $\alpha = 0$, $A \neq 0$; $\chi(t)$, an arbitrary function of t ; β , γ , constants expressed in terms of A , κ , ϵ ; s , entropy of the gas; M and λ , Mach and Khristianovich numbers; λ_* , subsonic velocity of the gas, corresponding to the critical velocity of the pseudogas; φ and ψ , velocity potential and the stream function; θ , angle of inclination of the velocity vector to the x axis; $P = 1/\lambda$, $Q = \rho_0/\rho_1\lambda$; \sqrt{K} and σ , coefficient and independent variable of the system of Chaplygin's equations. Indices: 1, pseudogas, i.e., the equilibrium mixture; ∞ , quantities in the unperturbed flow at infinity; 0, values at the point of stagnation of the flow; *, critical values of the quantities.

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STATIONARY EXCHANGE BETWEEN AN INFILTRATED GRANULAR BED AND A BODY IMMersed IN IT

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UDC 532.546:536.242

Stationary heat and mass outflow from a body in an infiltrated granular bed is studied taking into account the effect of the high-porosity zone near the surface of the body.

Problems concerning the stationary transfer of heat or mass from bodies placed in a filtrational flow were first posed and studied for bodies with a simple shape in [1, 2]. In [3] this formulation was extended to nonstationary transfer processes with absorption in the volume of the granular bed. Here the presence of a thin zone, in which the transfer coefficients differ considerably from their effective values outside it, on the surface of the immersed body was completely ignored. This is completely justified, if the characteristic size of the body is much greater than the structural size of the bed (diameter of the particles), and Peclet's number, constructed based on the size of the body, the filtration velocity, and the effective transfer coefficient, is not too large (see, for example, the experiments in [4]). When any of these conditions is violated, however, the existence of the indicated zone significantly changes the observed heat or mass flows compared to those determined theoretically neglecting this zone.

The idea of a layer of high thermal resistance near the surface of a body has been introduced repeatedly in different semiempirical variants of the theory and has been discussed in connection with the problem of external heat transfer in fluidized systems (see the review in [5, 6]). In application to exchange between bodies and filtration flows in stationary granular fills, it was recently used in [7, 8], where the zone near the wall was viewed as

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